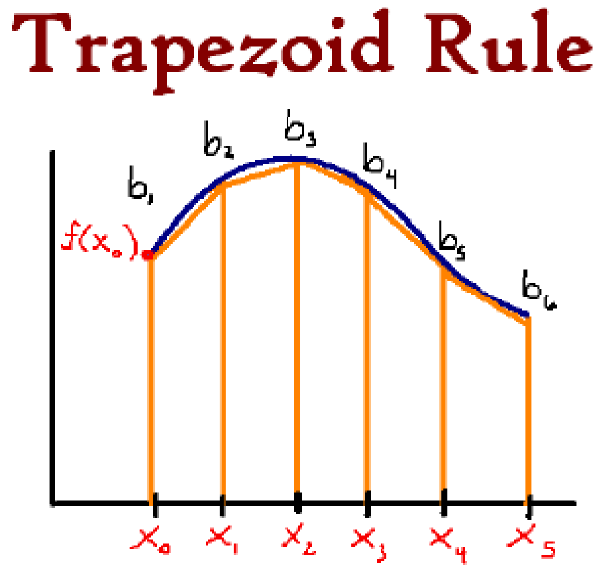
Unit 5: Trapezoidal Rule for Approximating Area under a Curve

Area of a Trapezoid = $\dfrac{(b_1+b_2)}{2} * h$

where,

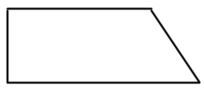
$h = \Delta x$

$b_1, b_2, ... = f(x_1), f(x_2)...$



$b_1 = f(x_1)$

$h = \Delta x$



$b_2 = f(x_2)$

Estimated area is the area of four trapezoids.

$$A = \frac{\Delta x}{2}[f(x_1)+f(x_2)] + \frac{\Delta x}{2}[f(x_2)+f(x_3)] + \frac{\Delta x}{2}[f(x_3)+f(x_4)] + \frac{\Delta x}{2}[f(x_4)+f(x_5)]$$

$$A = \frac{\Delta x}{2}[f(x_1)+f(x_2)+f(x_2)+f(x_3)+f(x_3)+f(x_4)+f(x_4)+f(x_5)]$$

$$A = \frac{\Delta x}{2}[f(x_1)+2f(x_2)+2f(x_3)+2f(x_4)+f(x_5)]$$

Generalize to any n, where n is the number of trapezoids.

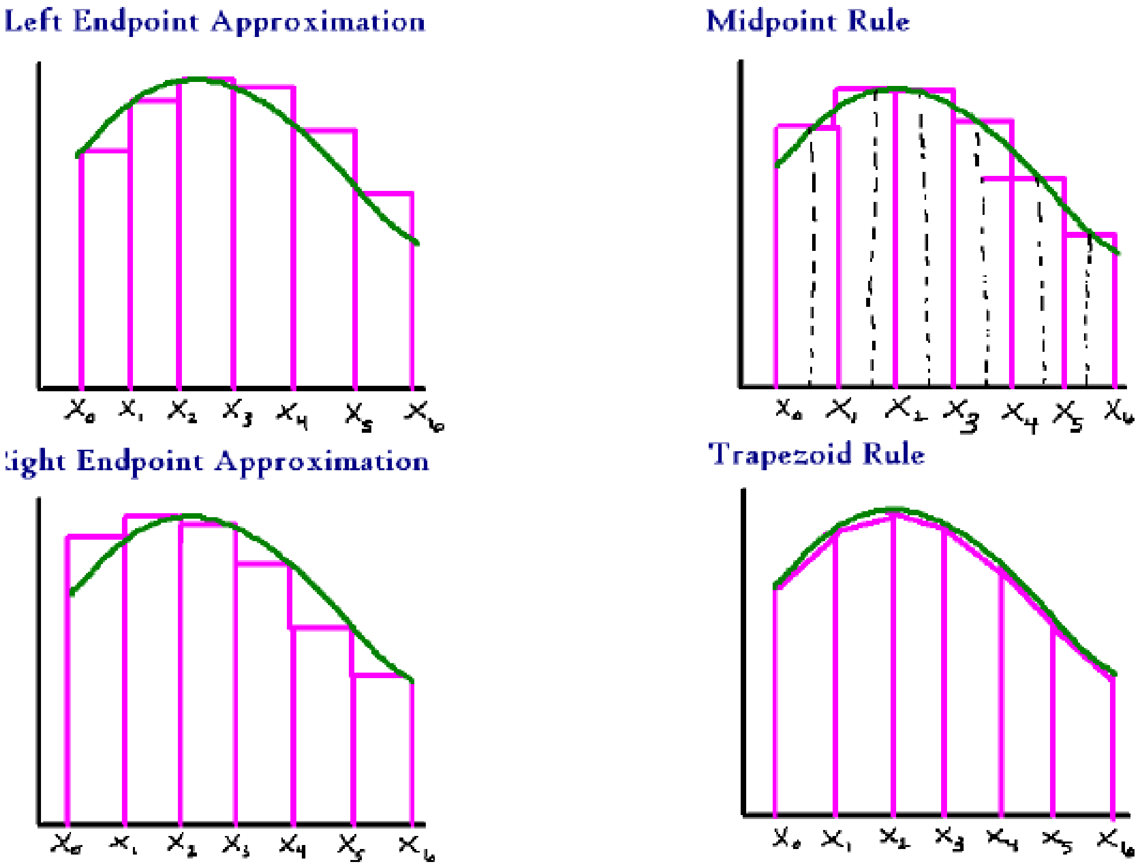
$$\boxed{A = \frac{\Delta x}{2}[f(x_1)+2f(x_2)+............+2f(x_n)+f(x_{n+1})]}$$

NOTE: The trapezoidal approximation is the same as the average of the left endpoint and the right endpoint approximation using rectangles.

Example:

$f(x) = -x^2 + 4x + 1$ in $[0, 4]$ with $n = 4$

Approximation Summary



Approximations with a Table of Values

The following chart indicates the speed of a sprinter during the first 10 seconds of the race.

Time (s)	0	1	3	5	6	9	10
Velocity (ft/s)	0	4.2	8.8	12.6	13.1	15.2	15.0

a) Estimate the distance traveled using a left-endpoint approximation method.

b) Estimate the distance traveled using a trapezoidal approximation method.